NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 35BAMS	**LEVEL:** 6
COURSE CODE: NUM701S	**COURSE NAME:** NUMERICAL METHODS 1
SESSION: JULY 2019	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

SUPPLEMENTARY/SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	Dr S.N. NEOSSI NGUETCHUE
MODERATOR:	Prof S.S. MOTSA

INSTRUCTIONS
1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations. All numerical results must be given using 5 decimals where necessary unless mentioned otherwise.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

Attachments

None

QUESTION 1 [36 Marks]

1.1. Given the function

$$f(x) = \ln(1+x)$$

1.1.1 Find the fifth-degree Taylor polynomial for $f(x)$ about $x_0 = 0$ ad use it to approximate f(0.2). [10]

1.1.2 Is the approximation in the previous question a good one? Justify your answer. [5]

1.1.3 Find a bound for the error in that approximation. [8]

1.2. Suppose that $g : [a,b] \to [a,b]$ is continuous on the real interval $[a,b]$ and is a contraction in the sense that there exists a constant $\lambda \in (0,1)$ such that

$$|g(x) - g(y)| \leq \lambda |x-y|, \text{ for all } x, y \in [a,b].$$

1.2.1 Prove that there exists a unique fixed point in $[a,b]$ and that the fixed point iteration $x_{k+1} = g(x_k)$ converges to it for any choice of $x_0 \in [a,b]$. [10]

1.2.2 Prove that the error is reduced by a factor of at least λ for each iteration to the next. [3]

QUESTION 2 [32 Marks]

2.1. Let $\{x_1, x_2, \ldots, x_n\}$ be n real and distinct interpolation nodes/points and V be the Vandermonde matrix

$$V = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \\ \vdots & \vdots & \cdots & \vdots \\ x_1^{n-1} & x_2^{n-1} & \cdots & x_n^{n-1} \end{pmatrix}$$

2.1.1 Let $L_i(x), i = 1, \ldots, n$ be the cardinal functions of the Lagrange interpolating polynomials. Use the fact that $L_i(x_k) = \delta_{ik}$ to show that V is non-singular. [7]

2.1.2 Let [10]

$$\Phi_n(x) = (x-x_1)(x-x_2)\cdots(x-x_n) = \sum_{j=1}^{n+1} a_j x^{j-1}.$$

Outline an algorithm for finding the entries of V^{-1} based on finding the coefficients of $L_i(x), i = 1, 2, \ldots, n$
(Hint: relate $L_i(x)$ to $q_i(x) = \Phi_n(x)/(x - x_i)$)

2.2. If using the following formula to compute an approximation of $f'(x)$:

$$f'(x) \approx \frac{1}{12\,h}\left[-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)\right],$$

2.2.1 find the order of convergence as $h \to 0$. [15]

QUESTION 3 [32 Marks]

3.1. Given the initial-value problem (IVP)

$$y' = y^2 + t^2, \quad y(0) = 2 \tag{1}$$

3.1.1 Write down in details the fourth-order Runge-Kutta (RK4) algorithm to solve the specific IVP given by Eq. (1). [10]

3.1.2 Consider the table given below and use the RK4 algorithm in the previous question to compute the missing values. Don't compute any result present in the table, use it when necessary. [22]

k	t_k	K_1	K_2	K_3	K_4	y_{k+1}
0	0.	4.0	4.6672		5.68345	2.38112
1	0.08		6.81695		8.70268	
2		8.68576	10.86571			3.85292
3		14.90257		21.68049	31.32096	
4	0.32		46.71911		100.62421	

END OF PAPER
TOTAL MARKS: 100

God bless you !!!